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White Paper

An Analysis of Pulling Tension Prediction in Large Radius Conduit Bends Typical of Directional Boring

Under-river directional bores or duct trenched along a highway often result in "gentle" curves that can be described as bends with large radii. When described in this manner, it is natural to approach calculating pulling tension in such sections using the "bend equations", but is this the appropriate approach?

The cable pulling bend equations assume the cable is being "pulled" to the inside of the bend. Then the normal component of the pulling force itself becomes the primary contributor to added frictional resistance and the required pulling force. These bend equations assume a form with the friction coefficient in an exponent. But, when the weight of the cable in the bend (gravitational force) is greater or even comparable to the pulling force, the cable will not ride on the inside of the bend, and the assumptions in the bend equations are not met.

Some example calculations will help us to better understand this limitation.

Long Horizontal Bend

Let's begin by looking at a long horizontal bend as shown in Figure 1 (top view).

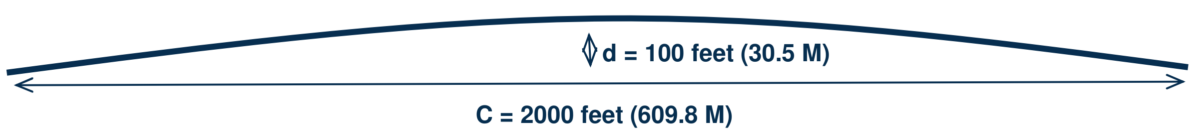

Figure 1

If we treat this as a large circular arc, and we know the chord (c = 2000 feet or 610 meters) and the maximum displacement (also called the chord height or sagitta) (d = 100 feet or 30.5 meters), we can use simple geometry to determine the radius (R) and angle of this conduit bend (θ). The radius (R) can be calculated using equation 1.

(1) $$R = \left(\frac{4d^2 + c^2}{8d}\right) = \left(\frac{40000 + 4000000}{800}\right) = \left(\frac{40400}{8}\right) = 5050 \text{ feet (1540 meters)}$$

In this case, the included angle of the arc is the same as the "bend angle", and is given by equation 2.

(2) $\theta = 2 \times \arcsin(\frac{c}{2r})$ = $2 \times \arcsin(\frac{2000}{10100})$ = 2×11.4 = 22.8 degrees.

So the conduit segment shown in Figure 1 can be described as a 22.8 degree conduit bend with a 5050 foot radius. From the radius and included angle, we can determine the length of the conduit arc is slightly greater than the chord at approximately 2010 feet (613 meters).

If we were pulling a cable that weighed 2 lbs/foot (2.97 kg/M) with 1000 lbf (454 kgf) back tension entering the bend, we could calculate with the "bend equations" to determine the pulling tension "add-on". The simplest bend equation is shown in equation 3. This equation assumes the tension coming into the bend is much, much greater than the weight of the cable in the bend, and this is clearly not true here. But it is instructional to do the calculation anyway.

(3) $T_{out} = T_{in} \times e^{\theta\mu}$

Remembering that θ must be in radians and assuming a friction coefficient (μ) of 0.2, equation 3 gives

$$T_{out} = T_{in} \times e^{\theta\mu} = 1000 \times e^{0.398 \times 0.2} = 1000 \times 1.083 = 1083 \text{ lbs}$$

a tension (out) of 1083 lbf (492 kgf or 4.82 kN), with an add-on of only 83 lbf (38 kgf). We can see from the form of equation (3) that a 1-foot radius bend would calculate the same as the 5050-foot radius bend. The calculation is clearly flawed by not including the weight of the cable.

Polywater's Pull-Planner™ 2000 software uses a horizontal bend equation that includes a weight factor. The equation for a single cable through a horizontal bend is given in equation 4*.

(4) $T_{out} = T_{in} \times \cosh(\theta\mu) + \sinh(\theta\mu) \times \sqrt{T_{in}^2 + (WR)^2}$

Calculating the example with equation 4 gives

$$T_{out} = 1000 \times \cosh(0.079) + \sinh(0.079) \times \sqrt{1000000 + (10100)^2} =$$

$$1000 \times 1.0032 + 0.07967 \times 10149.4 = 1003.2 + 808.6$$

= 1812 lbf (822.6 kgf or 8.06kN)

a result of 1812 lbf (822.6 kgf), or an add-on of 812 lbf (368.6 kgf) through the bend.

We can compare this to the calculation for a straight section of the arc length of 2010 feet (using equation 5).

(5) $$T_{out} = T_{in} + \mu WL = 1000 + 0.2 \times 2 \times 2010 = 1804 \text{ lbf (819 kgf)}$$

The calculation is simply the weight of the cable multiplied by the friction coefficient, and results in an add-on of 804 lbf (365 kgf). As expected, the calculation from equation 4 with a large radius bend is very close to an equivalent length straight section.

Long Vertical Concave-Up Bend

The bend equations* that include a vertical component are even more affected as gravity becomes the dominate force in large radius bends, and they do not predict tension accurately under these conditions.

For example, take the geometry of the bend described previously but make it a vertical bend in an under-river bore as shown in Figure 2 (side view).

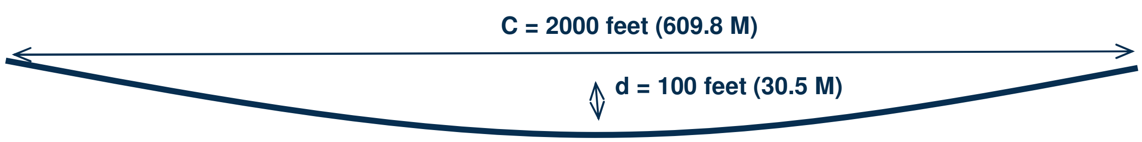

Figure 2

The vertical concave bend pulling equations are much like equation 3, but include a second term that adds or subtracts a complex gravitational term. Equation 6 is the common equation for pulling down through a concave bend and equation 7 for pulling up through a concave bend. Note that gravity is "pulling" the cable away form the inside of the bend, so in both cases the gravity factor is subtracted.

(6) $$T_{out} = T_{in} \times e^{\theta\mu} - \frac{WR}{1+\mu^2}\left[2\mu e^{\theta\mu}\sin\theta + (1-\mu^2)(1-e^{\theta\mu}\cos\theta)\right] \text{ (Concave pulling down)}$$

(7) $$T_{out} = T_{in} \times e^{\theta\mu} - \frac{WR}{1+\mu^2}\left[2\mu\sin\theta - (1-\mu^2)(e^{\theta\mu} - \cos\theta)\right] \text{ (Concave pulling up)}$$

From the symmetry, it is logical to calculate this bore as pulling "down" through an 11.4 degree concave bend (half the total bend of 22.8), followed by pulling "up" though an identical bend (equation 6 followed by equation 7).

So, for equation 6, with $e^{\theta\mu} = e^{.199 \times 0.2} = 1.0405$ we have

$$T_{out} = 1000 \times 1.0405 - \frac{10100}{1+0.04}\left[0.4 \times 1.0405 \times \sin(11.4) + (1-.04)(1-1.0405 \times \cos(11.4))\right]$$

$$= 1040.5 - 9711.5 \times [0.08227 + -0.02007] = 1040.5 - 604.6 = 435.9 \text{ lbf}$$

Taking the result into equation 7 we have

$$T_{out} = 435.9 \times 1.0405 - \frac{10100}{1+0.04}[2 \times 0.2 \times \sin(11.4) - (1-.04)(1.0405 - \cos(11.4))]$$

$$= 435.9 - 9711.5 \times [0.07906 - 0.05791] = 435.9 - 205.4 = 230.5 \text{ lbf (104.6 kgf)}$$

The result of the calculation is a counter intuitive reduction in predicted pulling tension of over 750 lbf (340 kgf). When a cable's gravitational weight pulls it away from the inside duct wall, the equations do not apply. The concave (or convex) vertical pulling equations should not used for large radius bends.

Alternative Calculation Approaches

As an alternative, we will calculate this segment as two inclined straight sections (of 1005 feet each). One will be going down at 11.4 degrees and the other coming up at the same angle. Equation 8 is the straight segment equation with an incline angle of ϕ.

(8) $T_{out} = T_{in} \pm WL(\sin\phi \pm \mu\cos\phi)$ (*+ for pulling up incline and – for down*)

or for the down pull

$$T_{out} = 1000 - 2 \times 1005 \times (\sin(11.4) - 0.2 \times \cos(11.4)) = 1000 - (2010 \times 0.00160)$$
$$= 996.8 \text{ lbf (452.5 kgf)}$$

and for the up pull

$$T_{out} = 996.8 + 2 \times 1005 \times (\sin(11.4) + 0.2 \times \cos(11.4)) = 1000 + (2010 \times 0.39371)$$
$$= 996.8 + 791.3 = 1788.1 \text{ lbf (811.8 kgf)}$$

The two inclined straight sections give an add-on of 788 lbs, and that moves to 870 lbs if we put a compensatory 22.8 degree bend at the bottom of the pull to make the direction change. We must, of course, ignore any sidewall pressure produced by this bend, since while the bend is real, its radius is not.

In the cited AEIC reference, there is an equation (9) given for a small angle vertical dip where T_{in} is less than RW.

(9) $T_{out} = T_{in} + \mu Wc$

We recognize this as the straight section equation 5, where the chord length has been substituted for the arc length. In our example, this is the difference between 2000 feet and 2010 feet, and we see that equation (9) would calculate an add-on of 800 lbf, less than a percent lower than the calculation treating the arc as a straight section, Another equation occasionally used for large radius bends is

(10) $$T_{out} = T_{in} + W\,\mathrm{R}e^{\theta\pi} - WR$$

In this case we see this calculates as

$$T_{out} = 1000 + 2\times5050\times1.083 - 2\times5050 = 1000 + 10938.3 - 10100$$
= 1838 lbf (834 kgf)

While all three of the above methods give similar results, at Polywater we use the method that treats the long vertical bore bends as inclined straight sections with a compensating small radius bend. To date, this has shown reasonable correlation with field pulling results

Please note the above analysis does not attempt to address the memory adjustments continuous duct may make in a large back-reamed borehole. Such adjustments must introduce an "element of bend" into the supposed straight sections.

**Various versions of the pulling equations have been published in a number of standards and cable installation literature. Many of the formulas here can be found in AEIC publication G5-90; Underground Extruded Power Cable Pulling Guide.*

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